Prove that each of the following languages is **not** regular.

1. $\left\{ 0^{2^n} \mid n \geq 0 \right\}$

2. $\{ 0^{2n} 1^n \mid n \geq 0 \}$

3. $\{ 0^m 1^n \mid m \neq 2n \}$

4. Strings over $\{0, 1\}$ where the number of 0s is exactly twice the number of 1s.

5. Strings of properly nested parentheses $()$, brackets $[]$, and braces $\{\}$. For example, the string $([]) \{\}$ is in this language, but the string $([)]$ is not, because the left and right delimiters don't match.

6. Strings of the form $w_1 \# w_2 \# \cdots \# w_n$ for some $n \geq 2$, where each substring w_i is a string in $\{0, 1\}^*$, and some pair of substrings w_i and w_j are equal.

Work on these later:

7. $\left\{ 0^{n^2} \mid n \geq 0 \right\}$

8. $\{ w \in (0 + 1)^* \mid w \text{ is the binary representation of a perfect square} \}$