"CS 374" Fall 2014 — Homework 7

Due Tuesday, October 28, 2014 at noon

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1. You are standing next to a water pond, and you have three empty jars. Each jar holds a positive integer number of gallons; the capacities of the three jars may or may not be different. You want one of the jars (which one doesn't matter) to contain exactly k gallons of water, for some integer k. You are only allowed to perform the following operations:

   (a) Fill a jar with water from the pond until the jar is full.

   (b) Empty a jar of water by pouring water into the pond.

   (c) Pour water from one jar to another, until either the first jar is empty or the second jar is full, whichever happens first.

   For example, suppose your jars hold 6, 10, and 15 gallons. Then you can put 13 gallons of water into the third jar in six steps:

   - Fill the third jar from the pond.
   - Fill the first jar from the third jar. (Now the third jar holds 9 gallons.)
   - Empty the first jar into the pond.
   - Fill the second jar from the pond.
   - Fill the first jar from the second jar. (Now the second jar holds 4 gallons.)
   - Empty the second jar into the third jar.

   Describe an efficient algorithm that finds the minimum number of operations required to obtain a jar containing exactly k gallons of water, or reports correctly that obtaining exactly k gallons of water is impossible, given the capacities of the three jars and a positive integer k as input. For example, given the four numbers $6, 10, 15$ and 13 as input, your algorithm should return the number 6 (for the sequence of operations listed above).

2. Consider a directed graph G, where each edge is colored either red, white, or blue. A walk[1] in G is called a *French flag walk* if its sequence of edge colors is red, white, blue, red, white, blue, and so on. More formally, a walk $v_0 \to v_1 \to \cdots \to v_k$ is a French flag path if, for every integer i, the edge $v_i \to v_{i+1}$ is red if $i \bmod 3 = 0$, white if $i \bmod 3 = 1$, and blue if $i \bmod 3 = 2$.

   Describe an efficient algorithm to find all vertices in a given edge-colored directed graph G that can be reached from a given vertex v through a French flag walk.

3. Suppose we are given a directed acyclic graph G where every edge e has a positive integer weight $w(e)$, along with two specific vertices s and t and a positive integer W.

   (a) Describe an efficient algorithm to find the *longest* path (meaning the largest number of edges) from s to t in G with total weight at most W. *[Hint: Use dynamic programming.]*

   (b) **[Extra credit]** Solve part (a) with a running time that does not depend on W.

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[1] Recall that a ***walk*** in G is a sequence of vertices $v_0 \to v_1 \to \cdots \to v_k$, such that $v_{i-1} \to v_i$ is an edge in G for every index i. A ***path*** is a walk in which no vertex appears more than once.

CS 374 Fall 2014 — Homework 7 Problem 1

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Describe an efficient algorithm that finds the minimum number of operations required to obtain a jar containing exactly k gallons of water, or reports correctly that obtaining exactly k gallons of water is impossible.

CS 374 Fall 2014 — Homework 7 Problem 2

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Describe an efficient algorithm to find all vertices in a given edge-colored directed graph G that can be reached from a given vertex v through a French flag walk.

CS 374 Fall 2014 — Homework 7 Problem 3

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(a) Describe an efficient algorithm to find the longest path from vertex s to vertex t in a dag G with total weight at most W.

(b) **[Extra credit]** Solve part (a) with a running time that does not depend on W.
