1. Prove that the expected space requirement of a skip list constructed on n numbers is $O(n)$.

2. Let S be a set of n points in the plane. A point p in S is called **maximal** (or *Pareto-optimal*) if no other point in S is both above and to the right of p. If each point in S is chosen independently and uniformly at random from the unit square $[0,1] \times [0,1]$ what is the *exact* expected number of Pareto-optimal points in S.

3. A *data stream* is an extremely long sequence of items that you can read only once. A data stream algorithm looks roughly like this:

   ```
   DoSomethingInteresting(stream S) :
   repeat
       x ← next item in S
       ⟨⟨ do something fast with x ⟩⟩
   until S ends
   return ⟨⟨ something ⟩⟩
   ```

   Describe and analyze an algorithm that chooses one element uniformly at random from a data stream, *without knowing the length of the stream in advance*. Your algorithm should spend $O(1)$ time per stream element and use $O(1)$ space (not counting the stream itself).